Meriam 6/5 : 2D Kinetics

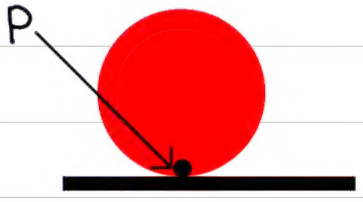
Equations for 2D kinetics

1/ $\sum \vec{F} = m\vec{a}_G$

2/ $\sum \vec{M}_O = I_O \vec{\alpha}$ +↺

3/ $\sum \vec{M}_G = I_G \vec{\alpha}$ +↺

4/ $\sum \vec{M}_P = I_P \vec{\alpha}$ +↺ for rolling wheel



$I_G = mk^2$, k = radius of gyration

$I_O = I_G + mr_{OG}^2$, parallel axis theorem.

These equations for kinetics must be combined with equations for kinematics to solve complex problems:

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$= \vec{v}_B + |\omega r_{AB}|$$

$|\omega r_{AB}|$ ⊥ to r_{AB}

where $$|\omega| = \frac{|\vec{v}_P - \vec{v}_Q|}{r_{PQ}}$$

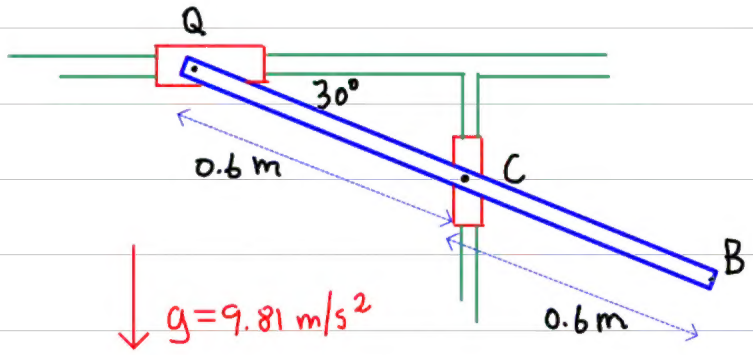
$$\vec{a}_A = \vec{a}_B + \omega^2 r_{AB}\hat{e}_n \; +/- \; |\alpha r_{AB}|\hat{e}_t$$

$\omega^2 r_{AB}\hat{e}_n$: vector pointing from A towards B

$|\alpha r_{AB}|\hat{e}_t$ ⊥ to r_{AB}

where $$|\alpha| = \frac{|\vec{a}_P - \vec{a}_Q|_t}{r_{PQ}}$$

E.g.



Given:

mass of blue rod = 4 kg

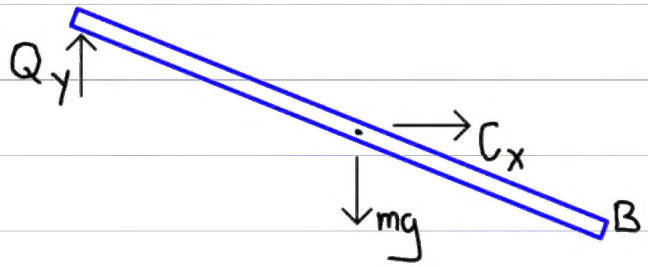
$V_0 = 0$

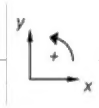
zero friction

C = center of gravity

Find α

1/ FBD, Define axes, +ve direction for rotation





2/ No fixed axis → (i) $\sum F_x = m(a_c)_x$

(ii) $\sum F_y = m(a_c)_y$

(iii) $\sum M_G = I_G \alpha$ +↺

(i) $\sum F_x = m(a_c)_x \rightarrow C_x = m(a_c)_x \rightarrow C_x = 0$

(ii) $\sum F_y = m(a_c)_y \rightarrow Q_y - mg = m(a_c)_y \rightarrow Q_y = 4(a_c)_y + 39.24$

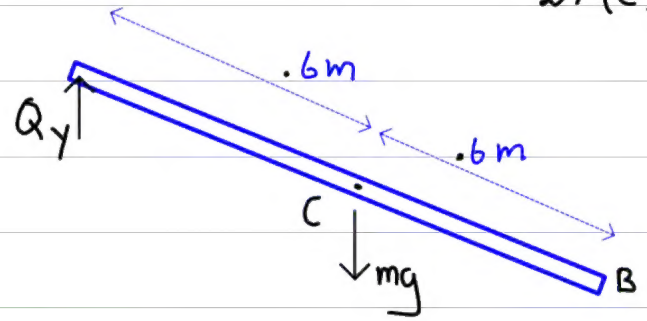
(iii) $\sum M_c = I_c \alpha$ +↺ $\rightarrow -(Q)_y(0.6\cos 30°) = \left(\frac{1}{12}\right)(4\text{kg})(1.2\text{m})^2 \alpha$

$\rightarrow Q_y = -0.924\alpha$

→ Two equations remain:

$$Q_y = 4(a_c)_y + 39.24 \quad (1)$$

$$Q_y = -0.924\alpha \quad (2)$$



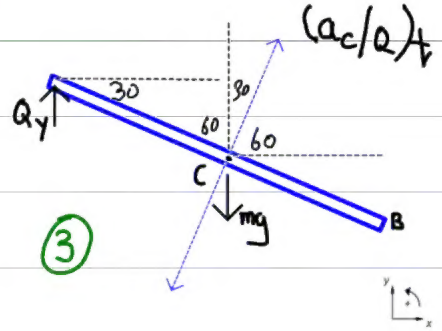
Three unknowns:

$(a_c)_y$, Q_y , α

→ Add a kinematic eqn to link unknowns together.

$$\vec{a}_c = \vec{a}_Q + (a_{c/Q})_n \hat{e}_n + (a_{c/Q})_t \hat{e}_t \quad \bigstar$$

($\omega^2 r_{c/Q} = 0$; $\alpha r_{c/Q}$)



→ Consider y-component of Eqn ★

$$(a_c)_y = 0 + 0 + (\alpha)(0.6\,m)(\cos 30^\circ) \quad (3)$$

Solve Eqns (1), (2) & (3) for α, $(a_c)_y$, Q_y

→

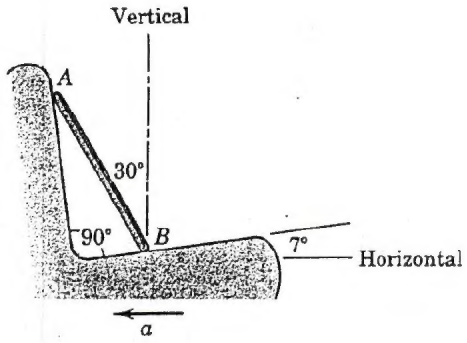
$$\alpha = -13.07\ s^{-2}$$

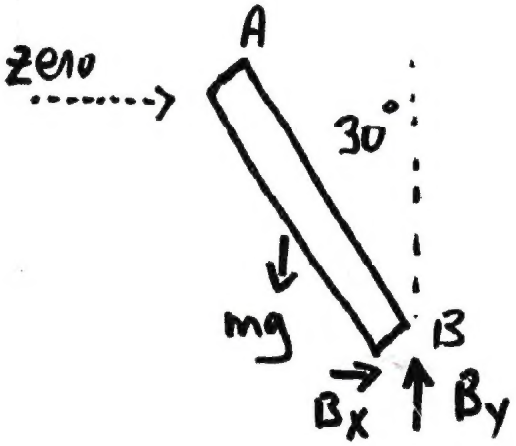
$$Q_y = 12.06 \text{ Newtons}$$

$$(a_c)_y = -6.8\ m/s^2$$

A uniform slender rod rests on a car seat as shown. Determine the deceleration a for which the rod will begin to tip forward. Assume that friction at B is sufficient to prevent slipping.

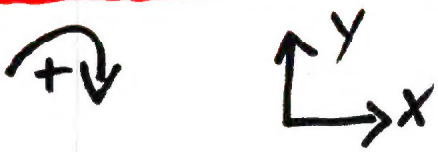
Ans. $a = 5.66 \text{ m/s}^2$





Note:

Force at "A" = 0 when rod starts to tip



1/ $\Sigma F_x = m(a_G)_x \Rightarrow$ $\boxed{B_x = m(a_G)_x}$

2/ $\Sigma F_y = m(a_G)_y \Rightarrow$ $\boxed{B_y - mg = 0}$

3/ $\Sigma M_{\circlearrowleft} = I_G \alpha \Rightarrow$ assume rod has length ℓ

$$-(B_x)\left(\frac{\ell}{2}\right)(\cos 30^\circ) - (B_y)\left(\frac{\ell}{2}\right)(\sin 30^\circ) = 0$$

$$\therefore \boxed{B_x \cos 30^\circ + B_y \sin 30^\circ = 0}$$

solve 3 eqns for $(a_G)_x$, B_x, B_y

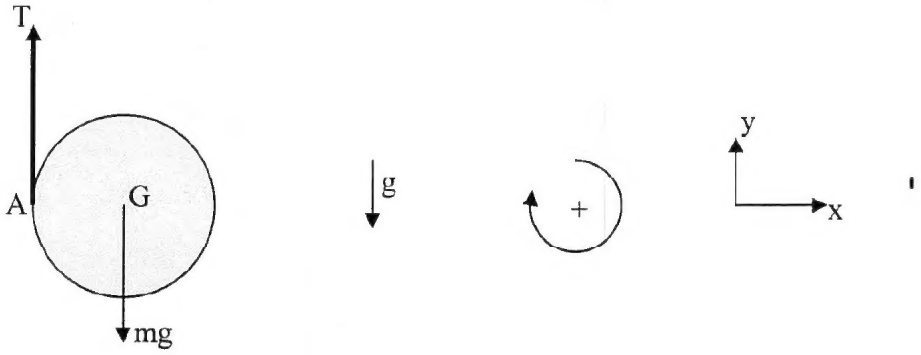
$$\Rightarrow (a_G)_x = -5.66 \text{ m/s}^2 \, \hat{i}$$

$B_x = -5.66\,m$ Newtons

$B_y = mg = 9.81\,m$ Newtons

Sample Problem: Planar Kinetics

A uniform disk of mass m and radius R has a rope would around it. The rope is being lifted upwards with a tension T at a speed v that is increasing by 3 m/s^2. Find the angular acceleration α of the disk, and the tension T in the rope.



Note: There is no stationary point "O" on this object.
Therefore, we **cannot** use the equation: $\Sigma M_O = I_O \alpha$

1/ $\Sigma M_G = I_G \alpha$ $\quad (T)(R) = \frac{1}{2} mR^2 \alpha$ (1)

2/ $\Sigma Fy = m (a_G)_y$ $\quad T - mg = m (a_G)_y$ (2)

3/ Kinematic relationship:

$$(a_{A/G})_t = \alpha R \quad \rightarrow \quad \alpha = (a_{A/G})_t / R = [3 - (a_G)_y]/R \quad (3)$$

Using these three equations, solve for T, α, and $(a_G)_y$ in terms of T, R, and m. Note that the speed v of the rope has no effect.

$$(a_G)_y = 1 - \frac{2g}{3}$$

$$T = m + mg/3$$

$$\alpha = 1 - 2g/3$$

27(f)

LAST Name: Answers

First Name: _____

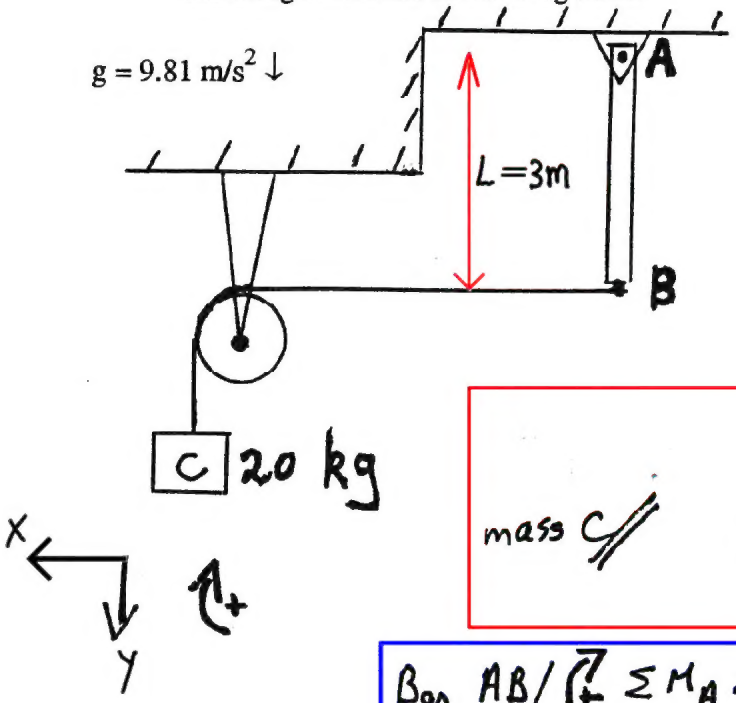
Student Number: _____

MIE 200F - Quiz number 7a - November 21/00
quiz duration = 25 minutes

A rope is tied between a 20-kg block and a thin rod of mass 10 kg and length 3 meters. The center of the wheel and the point "A" are both pinned to the ceiling. The 20-kg block is released from rest at the position shown.

(a) What is the moment of inertia of the rod about the point "A"?

(b) What will be the angular acceleration of the rod immediately following the release of the 20-kg block?



$T_2 = T_1 + U_{1\to 2}$

$I_{c\,of\,g}$ for a rod $= m L^2/12$ $\quad I_o = I_{c\,of\,g} + m d^2$

$I_{c\,of\,g}$ for a uniform disk $= m R^2/2$ $\quad I_{c\,of\,g} = mk^2$

$\vec{V}_P = \vec{V}_Q + \vec{V}_{P/Q}$ $\quad |\vec{V}_{P/Q}| = \omega R_{PQ}$

$\vec{a}_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha r)\hat{e}_\theta$ $\quad \vec{a}_P = \vec{a}_Q + \vec{a}_{P/Q}$

$T = \frac{1}{2} m (v_{c\,of\,g})^2 + \frac{1}{2} I_{c\,of\,g} \omega^2 = \frac{1}{2} I_o \omega^2$

mass C

$F = ma_c$

$(mg)_c - T = m_c a_c$

$T = -20 a_c + 196.2$ (1)

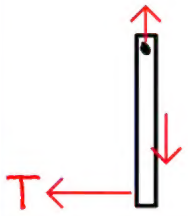
Bar AB / (+) $\Sigma M_A = I_A \alpha$

$3T = \left(\frac{1}{12} mL^2 + m(L/2)^2\right)\alpha$

$3T = \frac{1}{3} m_{AB} L^2 \alpha$

$3T = (\frac{1}{3})(10)(3^2)\alpha = 30\alpha$

$T = 10\alpha$ (2)



$I_A = 30\ kg \cdot m^2$

combine (1) & (2), using $a_B = a_c = 3\alpha$ (3)

$T = 10\alpha$

$T = -60\alpha + 196.2$ }

$10\alpha = -60\alpha + 196.2$

$\alpha = 2.80\ s^{-2}$ (+

$T = 28$ Newtons

$a_c = 8.4\ m/s^2$

Summary

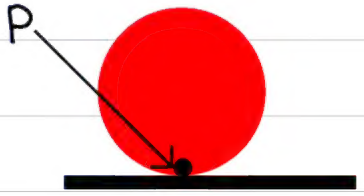
- 2D Kinetics – Sample problems.

1/ $\sum \vec{F} = m \vec{a}_G$

2/ $\sum \vec{M}_O = I_O \vec{\alpha}$ +↺

3/ $\sum \vec{M}_G = I_G \vec{\alpha}$ +↺

4/ $\sum \vec{M}_P = I_P \vec{\alpha}$ +↺ for rolling wheel



- When there are too many unknowns, combine these equations with kinematic formulas for relative acceleration:

$$\vec{a}_A = \vec{a}_B + \omega^2 r_{AB} \hat{e}_n +/- |\alpha r_{AB}| \hat{e}_t$$

$\hat{e}_n$: vector pointing from A towards B

$\hat{e}_t$: $\perp$ to r_{AB}

where $|\alpha| = \dfrac{|\vec{a}_P - \vec{a}_Q|_t}{r_{PQ}}$

Next Lecture

- Read Meriam 6/6: Work/Energy for 2D bodies